Closing today: HW_6A, 6B, 6C
Closing Wed: HW_7A, 7B, 7C
Midterm 2 is next Thursday, May 19th
Covers: 6.4, 6.5, 7.1-7.5, 7.7, 7.8, 8.1

Summary of 7.7: Two approx. methods
Here is what you do for n = 4 subdivisions:

1. Compute $\boldsymbol{\Delta x = \frac{b-a}{4}}$.
2. Label the tick marks: $\boldsymbol{x_i = a + i\Delta x}$

3. Trapezoid rule:

$$\frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + +2f(x_2) + 2f(x_3) + f(x_4)]$$

4. Simpson's rule:

$$\frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + +2f(x_2) + 4f(x_3) + f(x_4)]$$

Entry Task:
Approximate the following using n = 4 subdivisions and the Trapezoid and Simpson's Rules

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx$$

7.8 Improper Integrals

Motivation:

Consider the function $f(x) = \frac{1}{x^3}$.

Let's compute the following areas under this function.

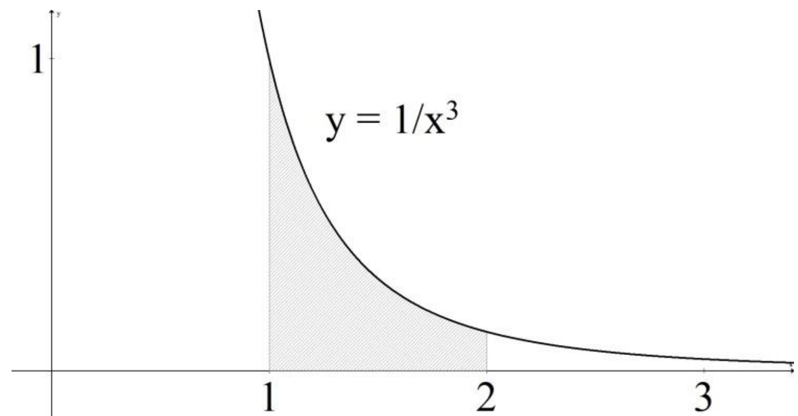
1. x = 1 to x = 2:

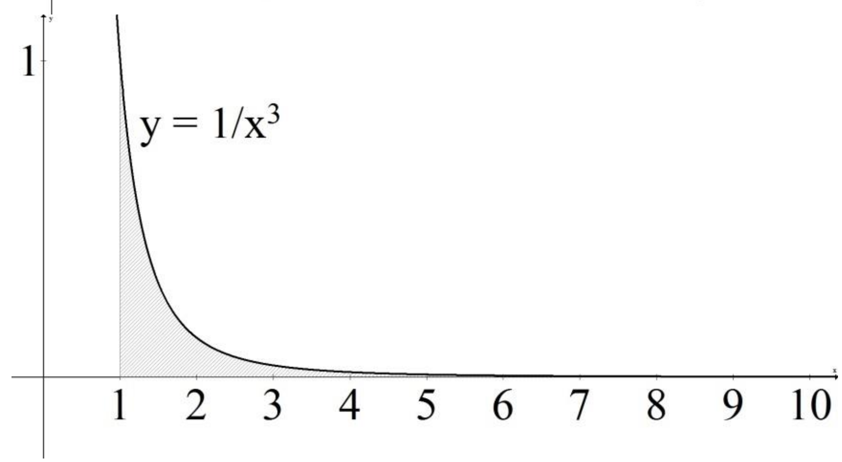
2. x = 1 to x = 10:

3. x = 1 to x = 100:

4. x = 1 to x = 1000:

5. x = 1 to x = t:





Limits Refresher

1. If stuck, plug in several values "near" t.
2. Know your basic values (and basic functions)

$$\lim_{t\to\infty} \frac{1}{t^a} = 0\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \frac{1}{e^{at}} = 0\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} t^a = \infty\,, \qquad if\ a > 0.$$

$$\lim_{t\to\infty} \ln(t) = \infty.$$

$$\lim_{t\to 0^+} \ln(t) = -\infty.$$

3. If you have an indeterminant form, you must use algebra and/or L'Hopital's rule

   Example:

$$\lim_{t\to\infty} \frac{\ln(t)}{t} \quad =$$

$$\lim_{t\to\infty} t^2 e^{-3t} =$$

A few general notes on **comparison**:
Suppose you have two functions f(x) and g(x) such that $0 \leq g(x) \leq f(x)$ for all values of x.

(a) If $\int_1^\infty f(x)dx$ converges,
then $\int_1^\infty g(x)dx$ converges.

(b) If $\int_1^\infty g(x)dx$ diverges,
then $\int_1^\infty f(x)dx$ diverges.

You can verify that

$$\int_1^\infty \frac{1}{x^p}\,dx\,, \qquad \text{converges for } p > 1.$$

$$\int_1^\infty e^{px}\,dx\,, \qquad \text{converges for } p < 0.$$